Research note: An Overview of the Traditional Discussion on Methodology of Quantitative Measurement of Economic Welfare

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Abstract

Degree of consumer satisfaction and economic welfare may fall under the category of subjective consumer psychology, and there is no way for a third party to directly observe or measure them quantitatively. Discussions about quantitative methodology of measuring changes in economic welfare using monetary scales have been going on for a long time. One of the representative examples is Dupuis and Marshall's consumer surplus. As for the theory of consumer surplus, significant operability has been pointed out as an advantage, as consumer surplus can be easily estimated directly from actual market data. On the other hand, these arguments faced many criticisms from a pure theoretical perspective. Later, as an alternative to consumer surplus, the concepts of Compensating Variation (CV) and Equivalent Variation (EV) were proposed by Sir John Richard Hicks. Although Hicksian CV and EV are said to have higher theoretical qualifications than Marshallian consumer surplus, they are based on Hicksian demand function, which is difficult to measure from actual market data. Therefore, it has been pointed out that it is seriously lacking in operability. This paper provides an overview of the traditional debate surrounding the quantitative methodology of the measurement of economic welfare.

Keywords: *Economic welfare,*
Hicksian compensated demand function,
Marshallian demand function,
Compensating Variantion, Equivalent Variation

1. Economic welfare measure

If the price system changes, which consumers face, the utility they receive from consumption will also change. This is because of changes in consumption behavior, such that consumers have to reduce the amount of a product they purchase due to an increase in price, or conversely, purchasing more than usual owing to decreasing in price. This means that there will be a change in the level of individual utility. In other words, a change in price causes a change in consumers' utility-maximizing behavior.

A change in consumer utility means a change in consumer satisfaction. In other words, this means changes in Economic Welfare. Now, the degree of quantitative change in consumer economic welfare brought about by price changes attracts our interests significantly. The discussion then moves on to what specific indicators can be used to measure changes in economic welfare, or in other words, what an appropriate welfare measure is.

Simply stating, it seems to be sufficient to take the difference in the utility level itself. However, in our discussion thus far, the utility level itself has meaning only in the hierarchy that contributes to comparison of magnitude, in other words, it has meaning only in ordinal utility. We have repeatedly confirmed that utility is meaningless. Therefore, in order to evaluate changes in economic welfare because of price changes, we have to find a new welfare measure to replace utility itself, which is hard for us to obtain in a visualized form.

Things like consumer satisfaction and economic welfare may fall into the category of subjective consumer psychology, and there is no way for a third party to directly observe or measure them quantitatively. Therefore, we should focus on money and develop an argument for finding a kind of objectivity as a welfare measure in so called as a monetary measure.

Discussions with respect to methodologies for measuring changes in economic welfare using monetary scales have been going on for a long time. One of the representative examples is Dupuis and Marshallian consumer surplus. Although it has been pointed out that it is easy to operate, such as the fact that consumer surplus can be estimated directly from actual market data, it has also been criticized from a theoretical perspective. Later, as an alternative to consumer surplus, Hicks proposed the concepts of Compensating Variation (CV) and Equivalent Variation (EV).

Although the concept of Hicksian CV and EV are said to have higher theoretical qualifications than Marshallian consumer surplus, they are based on Hicksian demand function, which is difficult to measure from actual market data. Therefore, it should be said that it may lack operability.

In this way, we will proceed with the discussion below, keeping in mind that Marshallian consumer surplus and Hicksian CV and EV both have advantages and disadvantages. First, let's talk about Hicksian CV and EV.

2. Hicksian Compensating Variation (CV) and Equivalent Variation (EV)

For the sake of simplicity, it is assumed that the consumer is faced with the choice of consuming two goods, and let the consumption quantities of each good be x^1 and x^2. Furthermore, regarding changes in consumption of goods because of price changes, we will focus only on good 2 in the following discussion. Regarding changes in price, we consider only good 2, the price of good 1 is fixed, and for convenience, we use relative price expression as "1". In other words, the price p, which is the slope of the budget line drawn on the consumption x^1, x^2 plane of the two goods, is the relative ratio of the prices of the two goods. p= (price of good 2) ÷ (price of good 1 (="1")) ≡ (price of good 2).

Consumption selection assumes two states and considers the change from state 0 to state1. That is, the price, utility level, consumption of good 2, and consumer income at the pre-change state 0 are p_0, U_0, x_0^2, and w_0 respectively. On the other hand, at the post-change state1, let the price, utility level, consumption of good 2, and consumer income be p_1, U_1, x_1^2, and w_1 respectively. Although the subscripts are complicated, Hicksian CV and EV strictly distinguish between the pre-change state and post-change state. Therefore, the meaning of the subscripts at pre-change state and post-change state are extremely important.

The first thing to be noted is that Compensating Variation (CV) and Equivalent Variation (EV) are concepts that attempt to measure changes in welfare as welfare measures based on changes in price when the utility level is assumed to be constant. As the two states change, the Compensating Variation (CV) corresponds to the pre-change utility level U_0, and the Equivalent Variation (EV) corresponds to the post-change utility level U_1. The definitions of Compensating Variation (CV) and Equivalent Variation (EV) are written in various ways depending on papers and microeconomics textbooks, however, Compensating Variation (CV) is defined as the amount of income that consumers are required to give up in order to offset the economic effect on consumers because of a change from pre-change state 0 to post-change state 1, and to keep the welfare level at state 0, which corresponds to pre-change welfare level. On the other hand, Equivalent Variation (EV) is the amount of income given to a consumer in order to make the welfare level at the post-change state 1.

Although the definition is extremely difficult to understand when written in words, expressing this as a utility function, for instance, the utility function U is a function of price p and income w, so the respective utility levels before and after the change are U_0 (p_0, w_0), U_1 (p_1, w_1), and the Compensating Variation (CV) and Equivalent Variation (EV) can be expressed as follows by using the utility level.

$$\begin{aligned} CV &: U_0 (p_0, w_0) = U_1 (p_1, w_1 - CV) \\ EV &: U_0 (p_0, w_0 + EV) = U_1 (p_1, w_1) \end{aligned} \tag{1}$$

To repeat, Compensating Variation is the income paid by consumers in order to keep them at the pre-change welfare level U_0. In other words, at the post-change welfare level, which corresponds to the post-change utility level U_1, CV is deliberately subtracted, and it is abandoned or paid from the income amount w_1 at the post-change state 1, and the utility level is artificially changed from U_1 to U_0, and the pre-change utility level can be obtained.

The Equivalent Variation is the amount of income paid to the consumer in order to achieve the post-change welfare level, which corresponds to the post-change utility level U_1, while the consumer remains at the pre-change welfare level, which corresponds to the pre-change utility level U_0. In other words, by intentionally adding EV to the income amount w_0 at the pre-change state 0, the post-change utility level change can be obtained from U_0 to U_1.

Above, we have added an explanation regarding the amount of income, but in reality, the analytical treatment of CV and EV differs greatly from each other depending on the point of time in which prices are considered. In other words, CV aims to achieve the welfare level, which is utility level U_0 at the pre-change state 0 under the price p_1 at the post-change state 1, while EV aims to achieve the welfare level, which corresponds to the post-change utility level U_1 under the price p_0 at the pre-change state 0. Above-mentioned explanation is showing on Figure 1, which is expressed by using an indifference curve and a budget line.

Something similar to the diagram used in Slutsky decomposition is found. Figure 1 depicts a budget line and a utility indifference curve on the consumption plane of two goods. As the price of the second good changes from the pre-change state 0 to the post-change state 1, the consumption

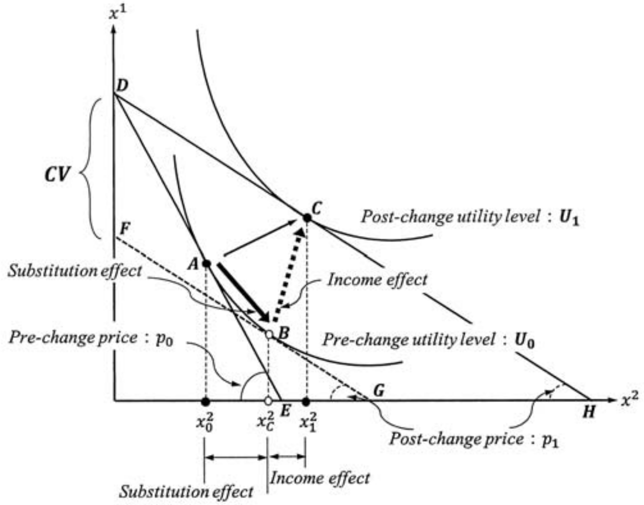

Figure 1　Conceptual diagram of Compensating Variation (CV)

point at which utility is maximized moves from A to C. The figure shows an example of a price drop, so the utility level increases from the pre-change utility U_0 to the post-change utility U_1.

Now, starts from the pre-change state 0. The budget line is indicated by the line DE on the diagram, and its slope is the price of good 2, which is p_0. In this case, the consumption point is A and the utility level is U_0. Now, as the state changes from here to state 1, the price of good 2 falls to p_1 and the budget line changes to the line DH. The slope is more gradual. Now, changes in consumption patterns due to price changes can be divided into two types: i.e. substitution effects and income effects using of the Slutsky decomposition, which has been adopted as a traditional discussion. First, a temporary budget line FG is drawn, which uses the post-change price p_1 without changing the pre-change utility level U_0, and then take the consumption point B which maximizes utility. This shift from point A to point B can be called a substitution effect, because the utility level remains constant and only the effect of the price change is taken. At this time, the change in the amount of demand of good 2 is changed from x_0^2 to x_C^2.

Next, we move the budget line upward in parallel while keeping the slope at the post-change price p_1 to obtain the consumption point C which touches the indifference curve corresponding to the post-change utility level U_1. The shift from point B to C during this period represents the effect when only income increases while the price remains constant at p_1. This corresponds to Slutsky's income effect. At this time, the change of the amount of demand of good 2 is changed from x_C^2 to x_1^2.

In this way, while holding the post-change price p_1 constant, the difference in income when giving up on reaching the post-change utility level U_1 and staying at the pre-change utility level U_0, which corresponds to DF on the diagram, is compensated. This corresponds to Compensating Variation (CV).

By replacing Figure 1 with a demand curve, the demand curve corresponding to the Compensating Variation is Hicksian compensated demand curve. The following Figure 2 shows a comparison between Hicksian compensaed demand curve and Marshallian demand curve.

Point A is the consumption point at the pre-change price p_0, utility level U_0, and at this point, Hicksian demand curve and Marshallian demand curve match with each other. As the price changes from p_0 to p_1 (in this example, the price falls), the displacement of the consumption point from

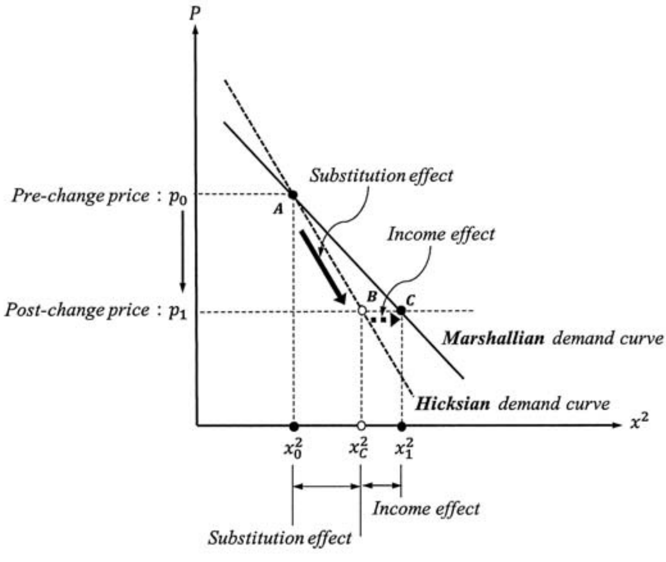

Figure 2 Hicksian demand curve corresponding to Compensating Variation

point A to B is due to the substitution effect on the Hicksian demand curve holding the utility level at U_0.

Now, the amount of demand of good 2 on the Hicksian demand curve and the Marshallian demand curve under the post-change price p_1 is compared as follows. On the Hicksian demand curve, which keeps the utility level at U_0 under the post-change price p_1, the amount of demand of good 2 is x_C^2, whereas on the Marshallian demand curve, it is x_1^2. The difference between x_C^2 and x_1^2 is just an income effect. In other words, on the Marshallian demand curve, the amount of demand increases by the amount which is expected to have an income effect on consumption on the Hicksian demand curve, and the utility level also increases from U_0 to U_1. Also, the income effect is expected at the post-change price p_1, then Marshallian demand curve has a gentler slope than Hicksian demand curve.

Next, let's look at the Equivalent Variation. Figure 3 shows the Equivalent Variation (EV) expressed by using an indifference curve and a budget line.

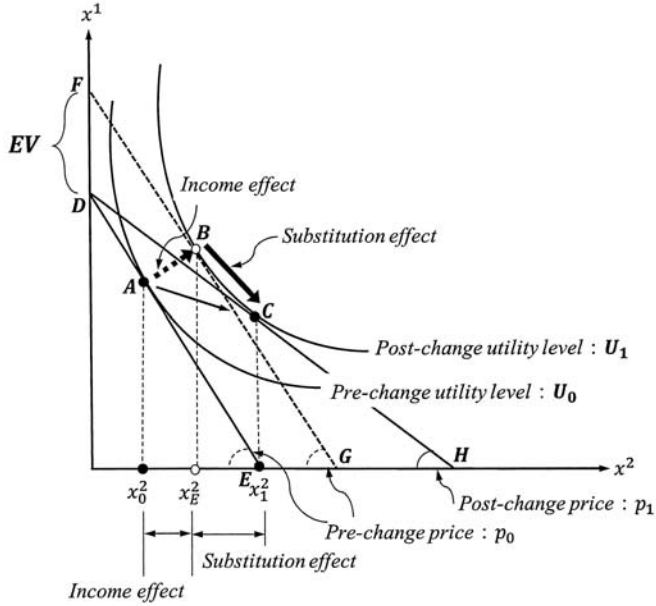

Figure 3 Conceptual diagram of Equivalent Variation(EV)

The Equivalent Variation was the additional amount of income required to achieve the post-change utility level U_1 at the given pre-change price p_0. First, consider consumption point A on the indifference curve corresponding to the pre-change utility level U_0, which is tangent to budget line DE at the pre-change price p_0. From here, the price is moved parallel along to the utility indifference curve of the post-change utility level U_1, keeping the pre-change price p_0, and the consumption point B which is tangent to it is obtained. The economic effect of moving from point A to point B corresponds to Slutsky's income effect. Furthermore, the movement from consumption point B to C is accompanied by a change in price from the pre-change price p_0 to the post-change price p_1 while keeping the utility level at U_1 which is the post-change utility level. This movement from point B to point C corresponds to the Slutsky's substitution effect. The Equivalent Variation (EV) corresponds to the amount of additional income required to shift the pre-change utility level U_0 to the post-change utility level U_1 while holding the price at the pre-change price p_0, and it is on the DF in the diagram.

Now, replacing Figure 3 with a demand curve as we did for the Compensating Variation, and take a look at Figure 4, which compares Hicksian compensated demand curve with Marshallian demand curve.

First, the Hicksian demand curve and the Marshallian demand curve

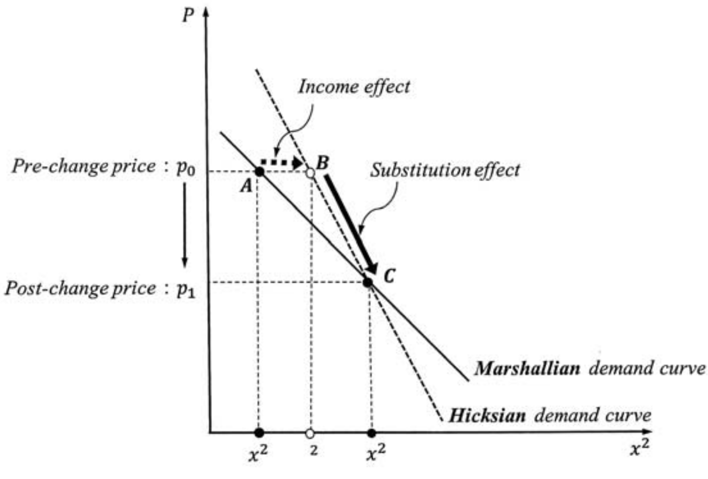

Figure 4　Hicksian compensated demand curve and Marshallian demand curve

coincide at the consumption point C under the post-change price p_1. The movement from consumption point B to consumption point C on the diagram corresponds to the substitution effect associated with the change in price from pre-change price p_0 to the post-change price p_1 (in this case, a price falls), and the utility level is fixed at the post-change utility level U_1. On the other hand, the difference between consumption point B on Hicksian demand curve (amount of consumption is x_E^2) and consumption point A on Marshallian demand curve (amount of consumption is x_0^2) at the pre-change price p_0 can be recognized. The deviation between x_E^2 and x_0^2 corresponds to the Slutsky's income effect.

The shape of the demand curve between Marshallian demand curve and Hicksian demand curve is compared with each other on the following Figure 5. The difference between two of them can be found in the income effect. In other words, the Marshallian demand curve takes into account the income effect in addition to the substitution effect, when the demand of good changes because of a change in price, whereas the Hicksian demand curve does not take into account the income effect. Therefore, the change in demand in response to a price change will cause a larger swing in Marshallian demand curve than in Hicksian demand curve, and the slope of Marshallian demand curve will become gentler owing to the income effect. Figure 5 shows Marshallian demand curve and Hicksian demand curve again. The Hicksian demand curve can be drawn as two lines, one corresponding to Compensated Variation, and the other corresponding to

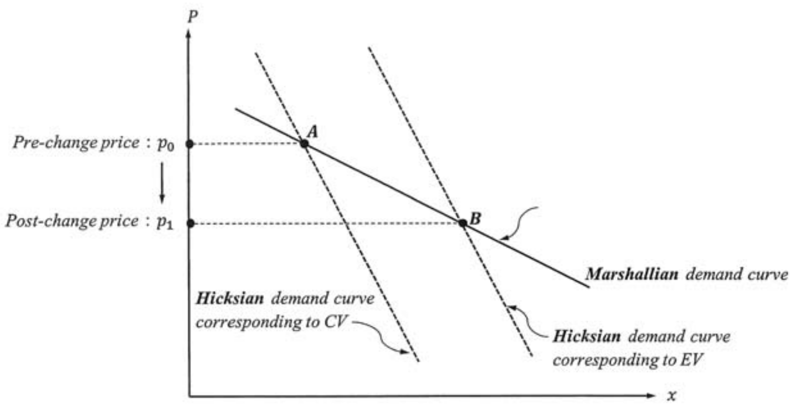

Figure 5　Hicksian demand curve corresponding to CV and EV

Equivalent Variation.

The Hicksian demand curve for the Compensated Variation intersects the Marshallian demand curve at point A at the pre-change price p_0, while the Hicksian demand curve for the Equivalent Variation intersects the Marshallian demand curve at point B at the post-change price p_1. Again, the deviation in the amount of demand at point A and B at the same price corresponds to the Slutsky's income effect.

3. Measuring methodology of Hicksian Compensating Variation / Equivalent Variation, and consumer surplus

Up to this point, there has been a conceptual discussion of Hicksian Compensating Variation (CV) and Equivalent Variation (EV). The question is what methodologies are appropriate to specifically measure these welfare measures. The expenditure function plays an important role in giving Hicksian Compensating Variation (CV) and Equivalent Variation (EV). The expenditure function is a function of the price vector p and the utility u. The utility u is a function of the consumption vector x. The expenditure function gives the minimum amount of expenditure necessary to achieve utility u under a price system p. Now, using this expenditure function, specific discussion as to the methodology of measuring Compensating Variation (CV) and Equivalent Variation (EV) is made here.

First, regarding Compensating Variation (CV), to restate the definition, Compensating Variation (CV) offsets the economic effect on consumers because of a change from the pre-change state 0 to the post-change state 1. It is defined as the amount of income that consumers are asked to give up in order to remain at the pre-change welfare level at state 0. In other words, the Compensating Variation (CV) is the amount of income. Let w_0 and w_1 be the income at the pre-change state 0 and the post-change state 1 respectively, and the minimum amount of expenditure required is equal to income w to obtain the maximized utility under price p and income w. Therefore, income w_0 and w_1 can be replaced using the expenditure function as follows.

$$w_0 = e\,(p_0, U_0),\ w_1 = e\,(p_1, U_1) \tag{2}$$

However, in Figure 3, the consumption point corresponding to the pre-change state 0 is A, and the consumption point corresponding to the post-change state 1 is C. The Compensating Variation (CV) can be defined as the deviation between the income amount at the pre-change utility level U_0 and the income amount at the post-change utiliy level U_1, while the price is fixed as the post-change price p_1. In other words, CV can be said as the difference in minimum expenditure. Therefore, the Compensating Variation (CV) must use the income amount at point B in the process of transitioning from consumption point A to C, rather than the difference between the income amount w_1 at the post-change state 1 and the income amount w_0 at the pre-change state 0. Letting this be w_B, it can be formulated as follows.

$$w_B = e\ (p_1, U_0) \tag{3}$$

The utility level is the pre-change utility level U_0, however, the reference price is the post-change price p_1. At this price p_1, the income amount can be equal to the minimum expenditure amount. By using this, the Compensating Variation (CV) can be shown as follows.

$$\begin{aligned} CV &= w_1 - w_B \\ &= e\ (p_1, U_1) - e\ (p_1, U_0) \end{aligned} \tag{4}$$

Now, the formula (4) can be transformed and rearranged a little bit technically as follows:

$$\begin{aligned} CV &= e\ (p_1, U_1) - e\ (p_1, U_0) \\ &= e\ (p_1, U_1) - e\ (p_0, U_0) + e\ (p_0, U_0) - e\ (p_1, U_0) \\ &= w_1 - w_0 + e\ (p_0, U_0) - e\ (p_1, U_0) \end{aligned} \tag{5}$$

Since at here, only price changes are considered and assumed as income does not change between the pre-change state 0 and the post-change state1, therefore $w_1 - w_0 = 0$, and equation (5) can be shown as follows:

$$CV = e\ (p_0, U_0) - e\ (p_1, U_0) \tag{6}$$

Paying attention to the subscript of equation (6), the utility level is constant at the pre-change utility level U_0, however, the price is embedded in the equation with pre-change price p_0 and post-change price p_1.

Next, looking at Equivalent Variation (EV) in the same way, Equivalent Variation (EV) can be defined as the amount of income given to a consumer in order to match the welfare level at the post-change state 1 with the welfare level at the pre-change state 0. According to Figure 3, the Equivalent Variation (EV) is the deviation between income amount corresponding to consumption point A at the pre-change state 0, and the income amount corresponding to consumption point B, which generates in the process of transitioning from state 0 to state 1. Therefore, the Equivalent Variation (EV) can be rearranged as follows, fixing the price p_0 at state 0.

$$\begin{aligned} EV &= w_B - w_0 \\ &= e(p_0, U_1) - e(p_0, U_0) \end{aligned} \tag{7}$$

Technically rearranging equation (7), the following equation can be obtained.

$$\begin{aligned} EV &= e(p_0, U_1) - e(p_0, U_0) \\ &= e(p_0, U_1) - e(p_1, U_1) + e(p_1, U_1) - e(p_0, U_0) \\ &= e(p_0, U_1) - e(p_1, U_1) + w_1 - w_0 \\ &= e(p_0, U_1) - e(p_1, U_1) \end{aligned} \tag{8}$$

Organizing this is as follows:

$$CV = e(p_0, U_0) - e(p_1, U_0) \tag{6'}$$

$$EV = e(p_0, U_1) - e(p_1, U_1) \tag{8'}$$

The CV and EV shown above can be rewritten in the form of integration, which integrate a very small amount of the expenditure function from the pre-change price p_0 to the post-change price p_1, while keeping the utility level constant. Then, the following equations can be obtained:

$$CV = e(p_0, U_0) - e(p_1, U_0) = -\int_{p_0}^{p_1} \frac{\partial e(p_0, U_0)}{\partial p} dp \tag{9}$$

$$EV = e\,(p_0,\,U_1) - e\,(p_1,\,U_1) = -\int_{p_0}^{p_1} \frac{\partial e\ (p_0,U_1)}{\partial p} dp$$

(10)

Now, take a close look at the above formula. The function form embedded in the integral sign is in the form of an expenditure function which is the partial differential of price. The form of this function is the compensated demand function, that is, the Hicksian demand function $h\ (p,\ U)$.

$$CV = -\int_{p_0}^{p_1} \frac{\partial e\,(p_0,U_0)}{\partial p} dp = -\int_{p_0}^{p_1} h\ (p,U_0)\,dp$$

(11)

$$EV = -\int_{p_0}^{p_1} \frac{\partial e\,(p_0,U_1)}{\partial p} dp = -\int_{p_0}^{p_1} h\ (p,U_1)\,dp$$

(12)

Applying a similar idea to Marshallian demand function $x\ (p,\ w)$, another welfare measure called Area Variation (AV) can be defined, which is similar to consumer surplus.

$$AV = -\int_{p_0}^{p_1} x\ (p,w)\,dp$$

(13)

Let's put CV, EV, and AV into Figure 6 as below. In Figure 6, two Hicksian demand curves corresponding to CV and EV respectively are shown, and one Marshallian demand curve with a gentle slope is shown in contrast to Hicksian demand curves.

The Compensating Variation (CV) shown in equation (11) is the value obtained by horizontally integrating the area from the pre-change price p_0 to the post-change price p_1, which is on the left side of the Hicksian demand curve corresponding to CV in Figure 6. This integrated value corresponds to the area of trapezoid p_0ADp_1. On the other hand, the Equivalent Variation (EV) is the value obtained by horizontally integrating the area to the left of the Hicksian demand curve (corresponding to EV) in Figure 6, from the pre-change price p_0 to the post-change price p_1. This integrated value corresponds to the area of the trapezoid p_0BCp_1.

Area Variation (AV) corresponds to the value obtained by horizontally integrating the area on the left side of Marshallian demand curve from the pre-change price p_0 to the post-change price p_1, which corresponds to the

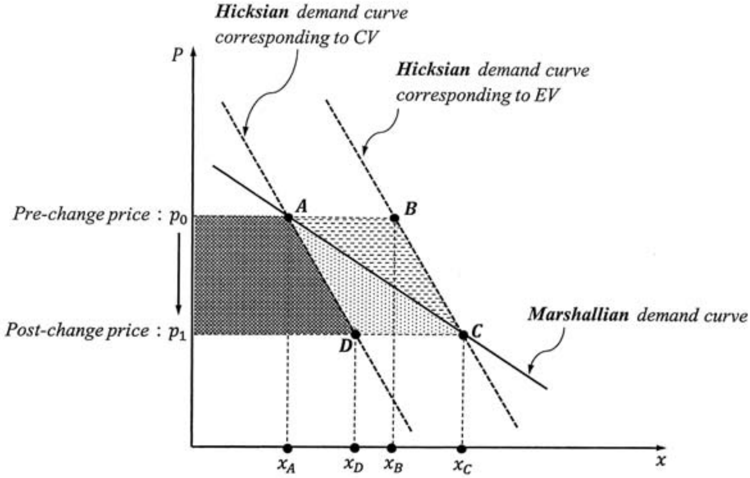

Figure 6 *CV*, *EV* and *AV*

area of trapezoid p_0ACDp_1. The discrepancy between Marshallian demand function and Hicksian demand function can be brought about owing to the existence of income effects. As Figure 6 shows, the difference among trapezoids p_0ADp_1, p_0BCp_1 and p_0ACDp_1 in trems of their size satisfies the following relationship:

$$EV > AV > CV \tag{14}$$

This size relationship holds true when the good is a superior good, that is, when a rise in price is inversely proportional to an increase in demand. In the case of lower-class goods and Giffen goods, this size relationship is reversed.

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